Name: Date: Period: 2

Indirect Proofs

Lesson Objective IWBAT write indirect proofs.

Suppose you are babysitting your neighbor's child, Billy, and their dog one afternoon after school. While you are watching Toy Story (aka the best Disney movie ever) with Billy in the family room, you hear a loud crash in the dining room - the dog knocked over and broke the family's expensive vase!

The parents come home from work and blame you, the babysitter, for breaking the vase. You explain your innocence to the parents as follows:

> "Let's suppose that I did break the vase.
>
> This means that I would have had to be in the dining room to break the vase, but I was in the family room watching Toy Story with Billy. Billy can verify that I was in the family room when the vase broke.
>
> If I broke the vase, I would have had to be in two places at once, which is impossible.
>
> I could not have broken the vase!"

Prove: didn't break vase

You would have used indirect reasoning to plead your case.

1. You started by making the temporary assumption that the desired conclusion was false… temp. assume: we did break vase
2. You then showed that this assumption lead to a logical impossibility… (aka contradiction)
3. So, what you are trying to prove MUST be true… we can't be in 2 diff. parts of house @ same time

How to Write an Indirect Proof

1. Identify the statement you want to prove.
2. Temporarily assume that this statement is false by assuming the opposite is true.
3. Reason logically until you reach a contradiction.
4. Point out that the desired conclusion must be true because the contradiction proves the temporary assumption false.

**Think "If F, then F ...then overall the statement is T. From Truth Tables!

Example:

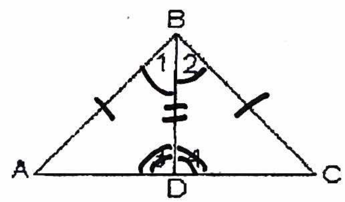
1. Write an indirect proof that in a given triangle, there can be at most one right angle.
   Given: ΔABC
   Prove: ΔABC can have at most one right angle.

Temporarily assume: ΔABC has at least 2 rt. $\angle$s. (F) By Δ sum Thm, $m\angle A + m\angle B + m\angle C = 180$. Let's say $\angle A$ and $\angle B$ are the rt. $\angle$s, so $90 + 90 + m\angle C = 180$. Then $m\angle C = 0$. But a Δ cannot have an $\angle$ measure of 0. $\rightarrow\leftarrow$ (F)
So, ΔABC can have at most one rt. $\angle$. (T)

2. Write an indirect proof that a scalene triangle cannot have two congruent angles.
Given: scalene triangle ABC
Prove: ΔABC cannot have two congruent angles.
HINT: Use the Converse of the Base Angles Theorem…

Temporarily assume: scalene $\triangle ABC$ can have 2 $\cong$ $\angle$s. Let's say $\angle A \cong \angle C$, then $\overline{AB} \cong \overline{CB}$ by $\triangle \rightarrow \triangle$. Then $\triangle ABC$ is isosceles because 2 sides are $\cong$. But we were given the $\triangle$ was scalene. $\rightarrow\leftarrow$ So, scalene $\triangle ABC$ cannot have 2 $\cong$ $\angle$s.

3. Write an indirect proof.
Given: $\overline{BD}$ bisects $\angle ABC$ and $\angle 3$ is not a right angle
Prove: $\overline{AB}$ is not congruent to $\overline{CB}$



Temp. assume $\overline{AB} \cong \overline{CB}$. $\angle 1 \cong \angle 2$ by def. of $\angle$ bisector (given.) Also, $\overline{BD} \cong \overline{BD}$ by reflex. prop $\cong$. So, $\triangle ABD \cong \triangle CBD$ by SAS $\cong$, and $\angle 3 \cong \angle 4$ by CPCTC. Since $\angle 3$ + $\angle 4$ form a linear pair, then they are also rt. $\angle$s. But we were given that $\angle 3$ is not a rt. $\angle$. $\rightarrow\leftarrow$ So, $\overline{AB}$ not $\cong$ to $\overline{CB}$.